# The Science of Juggling

## With Jeffrey Daymont

## Study Guide for Teachers

Introduction:
Juggling has always fascinated me.

After the challenge of learning to keep three balls in the air I wanted to do whatever I could to be a better juggler. In doing so I discovered how mathematics and physics applied to the juggling techniques I was learning. Even today I'm still fascinated by how the brain works and how it coordinates with the body to learn new skills.

The goal of this program is to use juggling to illustrate some basic principals of physics and mathematics to students, then provide a context for discussion afterwards. The level of discussion will vary depending on the level of your class and your curriculum. I've simplified some of the scientific terms to introduce them to younger students but you may want to be more specific for advanced students.

Finally I've included some activities to help students understand the process of learning. These are projects that will require a little time every day, but can have benefits throughout their education and beyond. Why do I think this is important? It is difficult to predict what skills the students of today will need for the jobs of tomorrow, but if you can teach your students HOW to learn they will be prepared for any future.

For more information about the program visit: [www.daymont.com/schools](http://www.daymont.com/schools)

Contents:

# Newton’s First Law of Motion

## Can be broken down into three parts:
## An object at rest will stay at rest
## An object in motion will stay in motion
## Until it is acted upon by an outside force

What does that mean?
“An object at rest will stay at rest”. If you set an object on a table it will not start moving on its own, instead it will continue to sit there -unless it is acted upon by an outside force.

What is an outside force?
An outside force is anything that could make that object move.
Set an object on a desk and ask the class ‘what outside force could make it move?’
(push it, blow on it, lift it...)

Once an object is in motion it will stay in motion
not only that it will keep the same speed and direction -**forever**
...until it is acted upon by an outside force.

If you were in outer space you could throw a ball and it would just keep going in a straight line forever until it hit something. What outside forces keep you from throwing a ball and having it fly forever?

Gravity- makes a ball fall to the ground
Friction- makes a rolling ball slow down and eventually stop
Wind Resistance- keeps you from throwing a ball of crumpled paper very far
The walls around you- will stop a ball from flying outside of the room.

Every moving object has both a speed and a direction, also called a <u>velocity</u>. An object <u>accelerates</u> whenever it changes velocity. That means that any time an object speeds up, slows down, or changes direction it is accelerating. According to Newton’s first law, the only way an object can accelerate is by an outside force.

Sometimes the outside force to change the path or speed of an object is you, like any time you throw or catch a ball. Even when you go for a walk you need an outside force. You need the friction against the floor to push against before you can go anywhere.

Every moving object also has <u>momentum</u>, which is a product of its velocity and mass. This just means that the heavier an object is, or the faster that it is going, the harder it will be to stop it.

Sir Isaac Newton

# What goes up must come down...

## Gravity: not just a good idea, it's the law.

Gravity is the force that pulls everything back down to earth. Gravity also keeps the moon going around the earth and the earth going around the sun. Luckily for jugglers (and rocket scientists), the force is always the same and predictable.

The equation to calculate the effect of gravity on earth is: $g=9.81\ m/s^2=32.2ft/s^2$.

In simple terms this means that if you drop an object it will continue to speed up as it falls. Not only that, it will speed up at the same rate, no matter how big or heavy the object is.

Also, if you throw an object up it will slow down at the same rate. That means it takes just as much time going up as does to come back down to where it started.

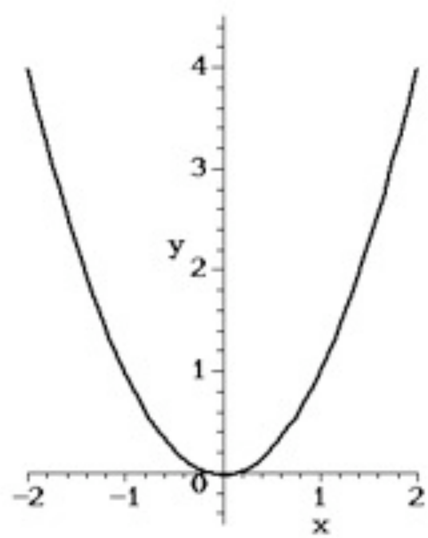
Of course you hardly ever throw something exactly straight up. Most objects travel through the air going across as well as up and down. The curved path is a shape called a parabola. A common place to see a parabola is the path water takes flowing from a water fountain.

The easiest way to draw a parabola is to plot the equation: $y=x^2$ (be sure to plot out negative values of x too!) You'll need to turn the curve upside down to see the path of a ball in flight.

It's important to notice that the curve is symmetrical, meaning both sides mirror the other. Jugglers can watch their throws at the top (apex) of the parabola and always know where it will land to catch it.

Another fun fact about the parabola is that it is a conic section. In other words, if you slice a cone with a plane parallel to an edge, the new curved edge will be a parabola. The other conic sections (circle, ellipse, hyperbola) all describe the paths that planets, comets, and satellites travel through space.

(Of course, these generalizations ignore the minor differences due to wind resistance, quantum and planetary scale, and that the forces of gravity are not exactly parallel, but if your students are ready to grasp these concepts, by all means go into more detail!)




Graph of $y=x^2$

# Keeping your balance

## Center of Gravity

Any time you balance an object you need to be aware of its center of gravity. Every rigid object has a center of gravity, or a center of mass. It's an imaginary center point in the exact middle of all of its weight. Any time you set an object down, it needs to be supported directly underneath this point or else it will tip over and fall.

How do you find a center of gravity?
The quickest way to estimate the location of an object's center of gravity is to suspend it from one point. The center of gravity will always hang directly below the point the object is hanging from.

You can balance a meter stick by laying it on your finger right at the center point (50cm). It will rest there because it is supported under its center of gravity. You can balance a broomstick the same way, but notice how the center of gravity is closer to the brush instead of the center.

Balancing a yardstick or a broom vertically takes a little more practice, but the principal is the same: you need to keep the bottom end underneath its center of gravity. This is true whether you balance it on your finger, you nose, or your toe. The trick is to watch the top of the object to see which direction it is falling, then move the bottom end underneath it. Remember, the higher the center of gravity the longer it will take to fall and the more time you will have to react! (It's much easier to balance a broom than a pencil.

When an object spins in space it will spin around its center of gravity, no matter which direction you spin it. If you hang an object from a string and spin it gently, the center of gravity will be directly beneath where it connects to the string.

Each of you have a center of gravity. Notice what happens when you lean from one side to another or shift your weight from one foot to the other. If your center of gravity ever goes past your feet, you will lose your balance and start to fall over.

If you are flying from a trapeze or diving from the high dive, no matter what gymnastics you do your center of gravity will still follow a perfect parabola. In fact, a high jumper doesn't need to get his center of gravity over a bar to get his body over it!

See related activity: Make a Holiday Mobile!

# What goes around comes around

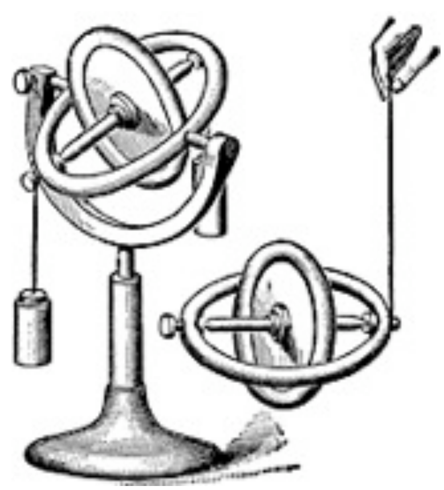
## spinning balls, tops, and gyroscopes

Is spinning a ball on your finger the same as balancing? Not quite. Instead it spins like a top or a gyroscope. Instead of just having linear momentum it has angular momentum and that helps keep it from falling.

What is Angular Momentum?
The principal of momentum goes back to Newton's First Law: An object in motion will stay in motion. Momentum is a combination of the weight (mass) of an object and the direction it is going in, so the heavier something is or the faster it is going, the more momentum it has. Angular momentum combines the speed an object is spinning and how far the weight is from the axis. You can increase the angular momentum by increasing the speed or moving the mass further from the axis.

What is an Axis?
The axis is the imaginary line that an object spins around. Spinning objects will continue to spin on their axis until acted upon by an outside force. Gyroscopes use this principle for navigation systems in space ships and rockets. The earth spins on an axis between the north and south pole to create night and day. The axis of a spinning object also goes through the center of gravity no matter which way you spin it.

How do you spin a ball on your finger?
First, it helps to have the right ball. Larger balls are more stable because their mass is further from the axis. Inflatable toy balls sold at Target or WalMart work fine -just let out enough air so you can hold it upside down with one hand. Basketballs are ok but you may need to use two hands to spin them.
Next you need to spin it on a vertical axis. Twist the ball into the air with a motion like you're screwing a lightbulb into the ceiling. I try to keep the seam of the ball horizontal with the floor. It won't matter how fast you spin it if the axis of the spin isn't vertical.
Finally you need to catch it on the tip of your finger. Aim to catch it on the axis -you will need to look for the spot in the middle of the spin.
If the spinning axis was vertical and you caught it just right, the ball should spin like a top on your finger. The spinning movement keeps the center of gravity right over your finger. The friction from your finger will eventually slow it down and it will fall.



A classic demonstration of Angular Momentum.
Sit a volunteer in an office chair that swivels. Have the volunteer hold small dumbbells out at arms length and spin the chair gently.
Notice how as the dumbbells are brought closer to the center the speed of the spin increases, then when the dumbbells are put out again the spin slows.
This demonstrates how as the velocity of the spin and distance of mass from the axis (moment of inertia) balance each other out to keep the angular momentum the same.

# Combinations and Permutations

## Combinations

Juggling boxes is all about switching places and changing their order. Before you can figure out all the tricks you need to figure out how many ways they can be rearranged.

Suppose you line up three pennies and each penny could be either heads or tails. How many different ways could you line them up? (I recommend giving your students time to work out their own answers first)

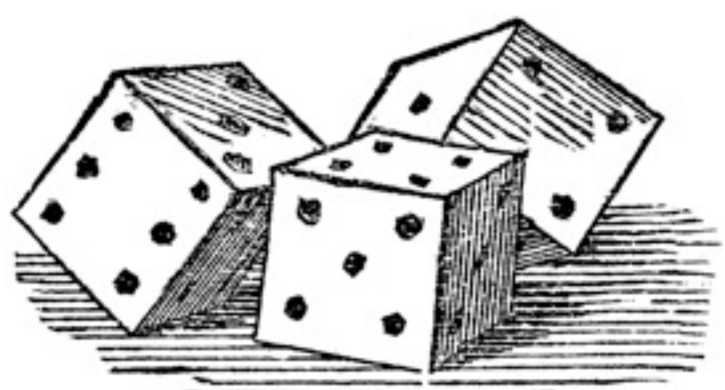
You can start with the obvious: one combination is all three heads (HHH)
then, if you have just one coin that is tails there are three places for that "tails" to be:
(THH, HTH, and HHT)
With two coins with "tails" you get three more combinations:
(TTH, THT, and HHT)
and finally you can have all three coins as tails (TTT).
That adds up to 8 different ways. That is also how many ways three boxes can be turned right side up or up side down in a juggling routine. You can turn just one box, two boxes, three boxes, or no boxes.

Now figuring out all of the possibilities and adding them up is fine when you just have three coins, but what if you had 10? or what if instead of coins with two sides you had dice with six sides?

Lets look at the three coins a different way:
There are only two things the first coin could be -heads or tails.
Then there are two things the second coin could be.
Two possibilities for the first coin times the two possibilities for the second coin is four
(2x2=4)
That times the two possibilities for the third coin comes out to 8 (4x2=8 or 2x2x2=8)
Now imagine if you added a fourth coin, how many combinations could you have?
2x2x2x2=16 combinations

You may recognize this as $2^4$. Every time you add another coin, there are two times as many possible combinations. So if you lined up 10 coins you have $2^{10}$ possible combinations of heads or tails, a total of 1024.

So what if you replaced the coins with dice? If each die has six sides, for each die you add you would multiply by 6. With 3 dice you have $6^3$ combinations (216) and with 5 you have $6^5$ combinations, or 7776 different possible results if you roll five dice.



# Combinations and Permutations

## Permutations

Permutations are a bit different than combinations. This time we will rearrange the orders of the coins without flipping them. How many ways can you line up a penny, nickel, and a dime (or the letters P,N,D)? (again, I recommend letting the students discover the answer on their own)

You should be able to find 6 different arrangements or permutations: PND, PDN, NPD, NDP, DPN, and DNP. It's no coincidence that that is the number of ways to switch the order of 3 juggling boxes (although it is a coincidence that the first four letters of "coincidence" spell "coin"!).

How do you keep track of them all though?
If you put the penny in the first position, you have only two places to put the nickel, and after that just one place left to put the dime. The same thing happens if you put the penny in the second or third position: you have only two places left to put the nickel which each leave just one place to put the dime.

You might notice a pattern if we try adding a 4th coin. If you have 4 possible places to put the first coin, that leaves 3 possible places for the second coin, 2 places for the third, and one place left for the last.

It turns out that you can calculate the number of permutations for three objects by multiplying 3x2x1=6. For 4 objects it is 4x3x2x1 or 24 total permutations.

So between turning the boxes and switching their places, they can be rearranged end to end 48 different ways ($3!x2^3$=6x8=48). And that's just the beginning.

Mathematicians express the factorial of a number as "n!" -in other words, the factorial of 5 is 5! or 5x4x3x2x1. These numbers are useful in advanced mathematics and in nature and can grow pretty fast. 5! is 120 and 7! is 5040. There are over 3 million ways you can rearrange 10 boxes (10!=3,628,800) and for 20 boxes it's over 2 million trillion!
(20!=2,432,902,008,176,640,0000)

Why would anyone need to think about numbers that are so big? Combinations and permutations help people calculate statistics for the lottery or mapping out genetic codes. I think it's more fun to ask how many different triple-dip ice cream cones you can make with 5 different types of ice cream? Or how many different pizzas are possible with eight different toppings? How do the numbers change if you allow duplicates? What if the order doesn't make a difference?

# Secrets of the Brain

## How the brain works and how you can learn anything

Science keeps learning more about how the brain works. A recent study at Oxford University shows that learning complex tasks like juggling grows new brain cells (both white and grey matter). I believe that the most important skill for a juggler is understanding how to learn something new.

Your brain learns by making connections and pathways between cells. When you do the same thing over and over, those pathways become stronger. When these pathways get strong enough you can do things without thinking about it. This is true if you are practicing a new skill, studying to learn new information, and even daily habits.

**Secret #1 You Learn as you Sleep!**
As you dream your brain reviews what you did that day and creates memories by making connections. Studies show that you can do better on a test if you study the night before and then sleep than if you stay up all night studying and not sleeping.

**Secret #2 It's not how many hours you practice, but how many days**
When you practice a skill day after day you brain builds on to the connections from before. You will learn more if you practice a little every day than if you practice all day once and then skip a few days.

**Secret #3 You can practice bad skills as well as good ones**
If you practice something for long enough you can do it without thinking about it. If you practice fancy basketball shots that always miss, you will get really good at doing fancy shots that always miss. Try to learn things the correct way first.

**Secret #4 You can rewire your brain!**
You can choose to change your habits if you want to. The longer you've been doing things one way, the more difficult it will be to make new pathways in your brain to start doing it a new way. Even after you make changes it will be tempting to go back to doing things the old way because those pathways are still strong in your mind. If you are patient and persistent you can trade old habits for better ones.

When you understand how your brain works you can learn anything. Knowing how you learn will help you get better at sports, get better grades, and make healthier choices in life. A good teacher or coach can guide you and show you what to learn, but only you can choose to make learning part of your daily routine.

"Give a man a fish and he will eat for a day,
Teach a man to fish and he will eat for a lifetime" -Chinese Proverb
"Teach a man to <u>learn</u> and he can achieve anything" -Jeffrey Daymont

# Extra credit questions

1. What is Newton's first law?
2. How do you accelerate an object?

3. What falls faster- a bowling ball or a golf ball?
What about a feather?

4. How high do you need to throw a ball for it to stay in the air for 1 second? 2 seconds? 3 seconds? 10 seconds?

5. What is harder to balance on one end- a meter stick or a pencil?
6. Why can't you touch your toes when your heels are against a wall?
7. Where is the center of gravity for a horse shoe?

8. Where is the axis on a spinning football?
9. Where is the axis of the earth?
10. What keeps a spinning basketball from falling off a finger?

11. If you flip a coin three times, what are the chances tit will come up the same way each time?
12. What are the chance is would come up tails every time?
13. If you roll 2 dice, what are the chances they will add up to 12? to 7?
14. If you roll 5 dice, what are the odds they will all come up the same? They will all be different?
15. If you have a combination lock with 30 numbers, how many different 3 number combinations are possible?
16. How many lottery tickets would you have to buy to guarantee you got the winning ticket? Assume you pick six numbers out of 50.

Questions for discussion:

Why are parabolas used in satellite dishes and headlights?

How hard was it to learn the alphabet?
What would be a good way to learn a new alphabet?

Try reading through the greek alphabet out loud every day. How long does it take for it to get easy?

What are some of the things you do every day?
Do you have any habits you want to change?
Is there something you want to be able to do or learn about on your own?

what kinds of things can you do without thinking about it? (tying your shoe, saying the alphabet, fixing a snack)

What are good habits that you could start?
(eating better, studying, practice an instrument or sport, getting a good nights sleep)

# Extra credit answers

1. An object at rest stays at rest and an object in motion will remain in motion unless acted upon by an outside force.
2. An outside force will accelerate an object

3. Both balls fall at the same speed.
A feather falls slower because it is slowed down by air. If you could remove the air from the room the feather would fall just as fast as the bowling ball.

4. 4.025 feet (one second), 16.1 feet (2 seconds), 36.225 feet (3 seconds), 1610 feet (10 seconds)

The equation for the distance an object falls in a given time is: $x = gt^2 / 2$
where g is the acceleration due to gravity (32.2 ft/sec$^2$) and t = time in seconds falling.
For example if a ball falls for 4 seconds, and we are calculating x as the distance:
$x = (32.2\ ft/s^2) * (4^2 s^2)/2 = 257.6$ feet

If you throw a ball to stay in the air for 1 second it will go up for one half second and fall for one half second. Using $x = gt^2 / 2$ where t = .5 seconds, the answer is 4.025 feet.

5. A meter stick has a higher center of balance making it easier to balance on one end.
6. As you lean forward, your center of gravity moves in front of your toes and you lose your balance.
7. The center of gravity for a horse shoe is somewhere in the middle ‘empty space’.

8. If you spin a football with a good spiral throw, the axis should be from one point to the other.
9. The axis goes from the north pole to the south pole.
10. Angular momentum

11. One in four times
12. One in eight times
13. One in 36 (for a 12), one in 6 (for a 7)
14. On average, one throw in 1296 will have all 5 numbers the same (yahtzee!) and five throws in 54 will have all dice come up differently.
15. If you allow numbers to be repeated, 27000. If all three numbers must be different, 24360
16. If you pick out 6 numbers from 50 possible choices where order doesn’t matter but there are no repetitions, there are 15,890,700 possible combinations.

The goal of the exercise with the Greek Alphabet is <u>not</u> about passing a test on the Greek Alphabet. Instead it is for each student to observe their own brain learning new information and discussing study habits. You can find the Greek alphabet on page 14.

Review “the Secrets of the Brain” on page 8. Help the students set up a system of reciting or writing the entire alphabet for just a few minutes every day in class over a period of a few weeks. Since there is no test or deadline, there is no need to study more than that. By reviewing the material daily and sleeping every night they will begin learning the material gradually. You may provide random or weekly quizzes that are not applied towards their grades but to help them observe their progress.

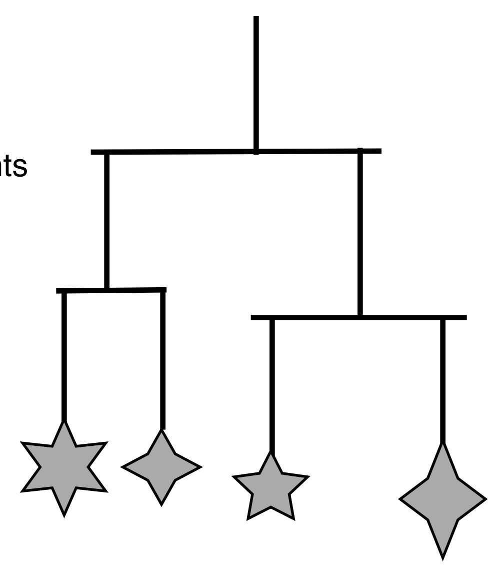
When they have all learned the alphabet sufficiently, discuss how the alphabet went from being difficult to easy over time. Did anyone prepare flash cards or other study aids to help? What else can you learn over time in the same way? If you had a test next week, what would be the best way to prepare?

# Activities

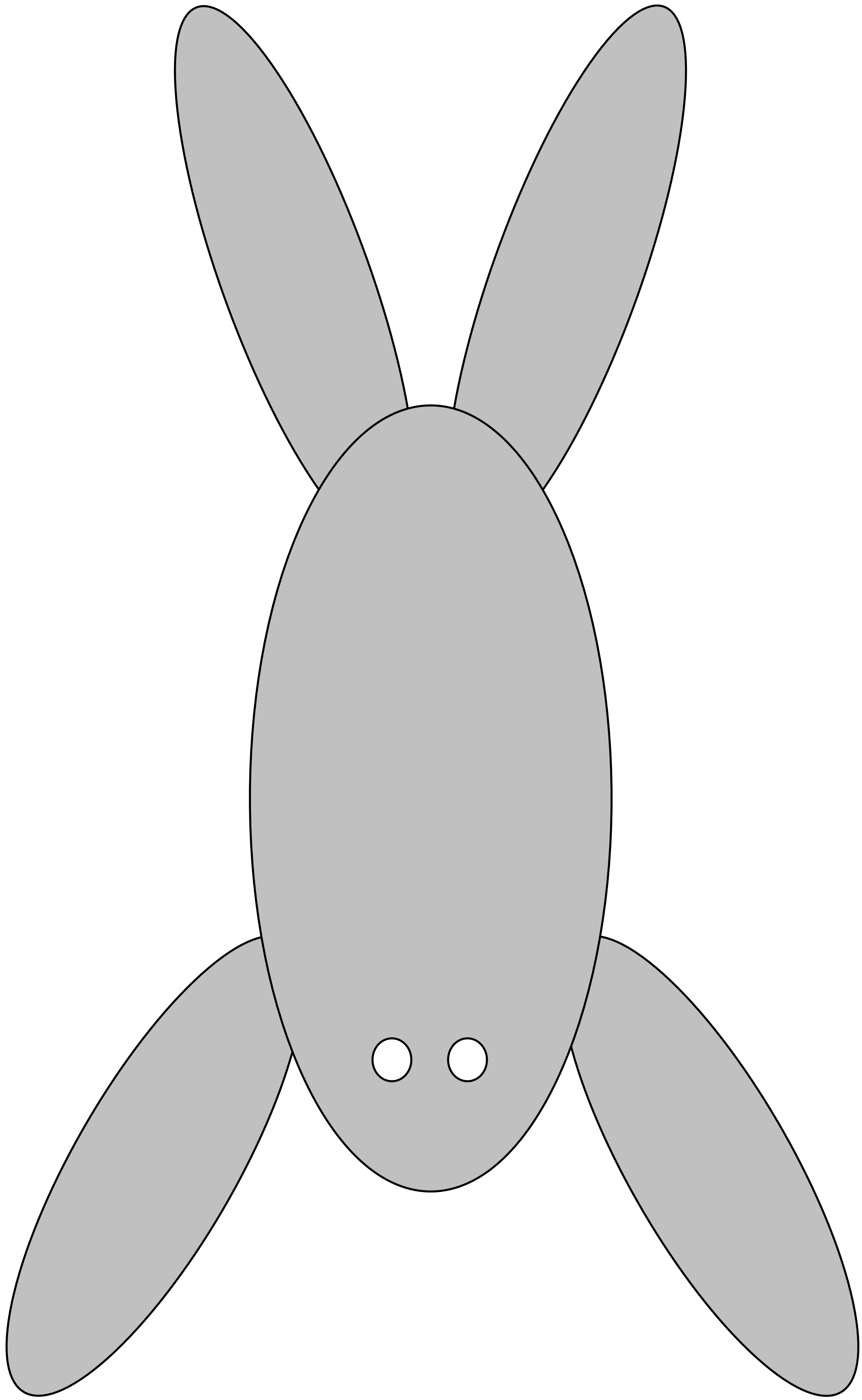
**Activity: Make a holiday mobile!**
Supplies:
Different lengths of sticks or stiff wire (about 6, 9, and 12 inches, can be cut from coat hangers. Use pliers to curl the wire ends into a small loop)
sturdy thread or string
small holiday ornaments with a place to attach string (slightly different sizes and weights are ideal, perhaps small crafts from a previous student project)
tape, scissors, and pliers

Step 1: tie or tape a short piece of string to each ornament (you should have about 4-6 inches of string after it is connected)
Step 2: tape or tie one ornament to each end of a short piece of wire (or stick)
Step 3: **Find the Center of Balance:** Tie a string to the middle of the wire and slide the knot until the wire hangs horizontally when suspended by the string. This point is the center of balance. Can you guess which end the center of balance will be closer to?
Step 4: Repeat steps 2 and 3 to make a second pair of ornaments. Then tie each pair to the ends of a medium sized piece of wire and find its center of balance.

Continue adding ornaments and sets of ornaments until the project is complete. Remember, the project doesn't need to be symmetrical! Try putting different numbers of ornaments on each end.



**Activity: The Balancing Frog!**
Print the frog on page 12 onto card stock or thin cardboard and carefully cut out.
Predict where the frog's center of gravity would be. Test your theories.
Attach paperclips or pennies to his hands or feet. How does that affect his center of gravity?
Can you make him balance on his nose?

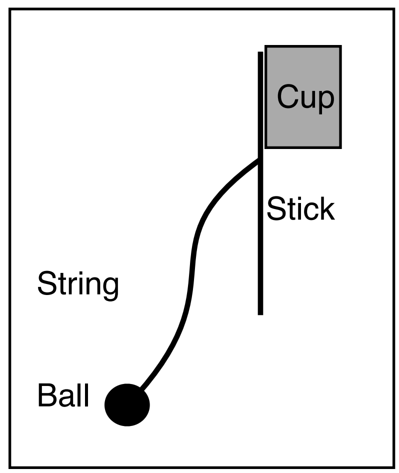


# The ball and cup toy

This is an activity with a variety of lessons.
First is the activity of making the toy.
Next, collecting data to keep track of improvement.
Finally, assembling the data to show whether learning actually happened.

Variations on the ball and cup toy have been found in different cultures around the world. Here is a simple way to make one.

You will need:
Popsicle sticks
Small dixie cups
String (about 12 inches per toy)
Tape
Small balls, heavy pompoms, or wooden beads that fit in the cups easily (about 1/2 to 1 inch diameter)




Using tape, attach the cup to the stick (open end up)
Attach the ball to the end of the string by taping or tying
Attach the other end of the string to the stick by the base of the cup by taping or tying (see diagram)

The game is to hold the stick in your hand then toss the ball up to catch it in the cup.

Collecting Data:
Have the students pair up and take turns collecting data.
Each tries ten times to catch the ball in the cup.
Their partner keeps track of how many times they succeed.
After two weeks draw a line graph to show their daily progress.

It is important that they know it is not a contest between themselves, but to measure improvement over time.  By trying the exercise every day they will be able to see how their mind improved from the first day.

Here are some tips for success:
Start with the ball hanging straight down and at rest: not swinging.
Try to lift the ball straight up into the air to about eye level.
When the ball reaches its apex (it stops going up and is about to fall down) bring the cup underneath the ball to catch it.

If you practice these tips you should see steady improvement!

| Greek | English | English | Latinized |
|---|---|---|---|
| **Letters** | **Names** | **Pronunciations** | **Transcriptio** |
| | | | |
| Αα | **alpha** | AL fuh | a |
| Ββ | **beta** | BAY tuh | b |
| Γγ | **gamma** | GAM muh | g |
| Δδ | **delta** | DEL tuh | d |
| Εε | **epsilon** | EP suh luhn | e |
| Ζζ | **zeta** | ZAY tuh, ZEE tuh | z |
| Ηη | **eta** | AY tuh, EE tuh | e |
| Θθ | **theta** | THAY tuh, THEE tuh | th |
| Ιι | **iota** | igh OH tuh | i |
| Κκ | **kappa** | KAP uh | k, c |
| Λλ | **lambda** | LAM duh | l |
| Μμ | **mu** | MYOO, MOO | m |
| Νν | **nu** | NOO, NYOO | n |
| Ξξ | **xi** | KSIGH, ZIGH, SIGH | x |
| Οο | **omicron** | AHM i krahn", OH mi krahn" | "short" o |
| Ππ | **pi** | PIGH | p |
| Ρρ | **rho** | ROH | r; rr; rh |
| Σσς | **sigma** | SIG muh | s |
| Ττ | **tau** | TAW, TOH | t |
| Υυ | **upsilon** | UP suh lahn", YOOP suh lahn" | y or u |
| Φφ | **phi** | FIGH | ph |
| Χχ | **chi** | KIGH | ch |
| Ψψ | **psi** | SIGH, PSIGH | ps |
| Ωω | **omega** | oh MEG uh, oh MAY guh | "long" o |